



Optimization Techniques In Financial Portfolio Management: A Mathematical Perspective

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Abstract

Traditional portfolio optimization models, such as the mean-variance framework, often fall short in addressing uncertainty, parameter estimation errors, and nonlinear investor objectives. This study provides a unified mathematical framework incorporating convex, stochastic, robust, and multi-objective optimization methods to model financial portfolios with greater theoretical rigor. The paper derives analytical conditions for the existence, uniqueness, and sensitivity of optimal solutions across formulations. A simplified symbolic example with three assets demonstrates the practical implications: increasing the risk aversion coefficient from 1 to 10 resulted in a 40% reduction in portfolio volatility, accompanied by a 15% decrease in expected return. This shift reflects a predictable yet mathematically tractable trade-off between risk minimization and return sacrifice, as governed by parameter tuning within each optimization model. The study underscores the value of mathematical generalization for improving the realism and robustness of portfolio models without relying on empirical backtesting. The comparison of model behaviors provides deep insight into the structural flexibility needed for informed financial decision-making under uncertainty. Theoretical contributions include formal derivations, comparative complexity analysis, and guidance for future integration with intelligent adaptive systems and algorithmic frameworks.

Keywords: portfolio optimization, convex optimization, stochastic modeling, robust optimization, mathematical finance, risk minimization

1. Introduction

One of the most important uses of mathematical optimization in contemporary finance is still portfolio management. The overall objective is to identify asset allocation policies that balance between the expected return and the risk associated with them. Ever since Markowitz proposed the mean-variance maximization model, mathematical models have been central to portfolio theory. The discipline has developed over time to be based on both linear and quadratic programming, as well as stochastic, robust, and distributionally robust optimization methods to enhance resilience in the presence of uncertainty (Senescall and Low, 2024; Platanakis et al., 2021).

The early models, though they form the basis of the various models, had strong assumptions like normally distributed returns, known covariances, and investor rationality. CAPM and its extensions assisted in the risk-

adjusted valuation, yet they relied on unrealistic assumptions (Blanchet et al., 2022; Lotfi and Zenios, 2018). Contrarily, new methods are aimed at solving real-world anomalies like tail risk, asymmetric distribution, and incomplete information. All these problems require optimization schemes that are not just computational acceptable, but are also theoretically acceptable in the face of ambiguity.

To handle uncertain probability distributions, stochastic dominance and empirical likelihood frameworks have been developed to deal with decision-making (Post et al., 2018). Liesiö et al. (2020) also build on this argument by suggesting portfolio diversification methods when the probability information is incomplete, since market dynamics are not necessarily entirely quantifiable or predictable. Equally, the use of robust optimization has gained popularity due to its capacity to provide stable results in the worst-case scenario. Bertsimas et al. (2018)

developed robust data-driven models based on the learning of past data combined with worst-case reasoning to construct portfolios that can be sustained under model ambiguity.

Nevertheless, as the attractiveness of robust and stochastic optimization is on the rise, most of the formulations are not easily analyzed mathematically. Indicatively, Caccioli et al. (2018) emphasized that optimization of portfolios in the context of expected shortfall is quite challenging, and the error in estimation may result in the creation of a highly unstable model. Georgantas et al. (2024) also compared the robust optimization techniques and found discrepancies in performance and traceability between the techniques. An integrated framework is yet to be achieved, particularly when incorporating robust and stochastic models in one consistent mathematical view.

Diversification logic is also another issue. Hwang et al. (2018) compared naive and optimized diversification strategies, highlighting the concealed tail risks of naive allocation strategies. In the meantime, Kim et al. (2018) showed the superiority of well-designed equity portfolio designs over classical methods through the consideration of both estimation risk and returns variance. The developments demonstrate an increasing realization of the nonlinear risk structures that its traditional approaches might not be able to quantify.

Regardless of these improvements, a lot of the models suggested over the past few years continue to be poorly developed. Because robust portfolio optimization does not have a consistent taxonomy in mathematics, as it was observed in the categorized review by Xidonas et al. (2020), there is no easy way to compare the methods applied to solve a problem of one type with another. Also, existing research typically optimizes within a given paradigm, either stochastic, robust, or heuristic, but does not provide a synthesis or cross-paradigm comparison of the models. This constrains generalizability as well as theoretical insight.

Given these concerns, a coherent mathematical approach to optimization methods in portfolio management that combines different models, evaluates their theoretical assumptions, and provides a standardized framework in which the quality and feasibility of solutions can be measured is urgently required. This kind of treatment would not only make the portfolio theory more scientific but would also inform the creation of more resilient financial strategies.

Objectives

1. To present a mathematically integrated and comparative framework of optimization techniques used in financial portfolio management, including convex, stochastic, and robust models.
2. To explore the theoretical underpinnings of these models by deriving existence, uniqueness, and stability conditions for optimal portfolio solutions.

2. Mathematical Formulation of Portfolio Optimization

2.1 Model Assumptions

Let there be a finite set of n financial assets, indexed by $i = 1, 2, \dots, n$. Each asset has an associated random return, r_i , and the investor selects portfolio weights w_i , representing the proportion of total capital invested in the asset i . The primary objective of portfolio optimization is to determine the optimal weight vector, $\mathbf{w} = (w_1, w_2, \dots, w_n)$ that maximizes expected return while minimizing risk.

The following standard assumptions are considered:

- Non-negativity constraint: $w_i \geq 0$ for all i , assuming no short-selling.
- Budget constraint: $\sum_{i=1}^n w_i = 1$, ensuring full capital allocation.
- Return model: Returns r_i are modeled as either random variables (stochastic models) or values within uncertainty sets (robust models).
- Risk measures: May include variance, value-at-risk (VaR), conditional value-at-risk (CVaR), or sparsity penalties for parsimony.

As Zhao et al. (2023) note, modern portfolio design increasingly incorporates constraints that promote sparsity and account for estimation errors under uncertainty. These considerations shape the formal optimization problem's structure and solution approach.

2.2 General Optimization Problem

A general portfolio optimization problem is defined as follows:

$$\min_{\mathbf{w}} F(\mathbf{w}) = \phi(\text{Risk}(\mathbf{w}), \text{Return}(\mathbf{w}))$$

Subject to:

- $\sum_{i=1}^n w_i = 1$
- $w_i \geq 0, \forall i$

The function $\phi(\cdot)$ represents a decision-maker's preference over the trade-off between risk and return. In a multi-objective context, this may be treated as a scalarization of competing objectives. As noted by Takano and Gotoh (2023), a unified framework for treating these formulations mathematically allows analysts to interpret classic and modern approaches within a common optimization landscape.

2.3 Specific Optimization Frameworks

Convex Optimization

In cases where the objective as well as the feasible region is convex, the portfolio optimization problem guarantees global optimality. Convex formulations allow easily analyzed analysis, especially when risk is modeled as a quadratic or piecewise-linear. Bertsimas and Cory-Wright (2022) stress the importance of convex and sparse formulations to obtain the scalable models of portfolios that can be computed even when the asset universe is very large.

Quadratic Programming

The classical Markowitz model is typically expressed as a quadratic programming (QP) problem:

$$\min_{\mathbf{w}} \mathbf{w}^T \Sigma \mathbf{w} - \lambda \mathbf{w}^T \boldsymbol{\mu}$$

where Σ is the covariance matrix of asset returns, $\boldsymbol{\mu}$ is the vector of expected returns, and λ is the risk aversion parameter. The structure of this formulation ensures

convexity, but sensitivity to parameter estimation limits practical robustness.

Stochastic Optimization

Stochastic models are returns that are considered random variables with known or partially known distributions. The goal usually tends to reduce the anticipated loss or maximize the anticipated utility subject to probabilistic restrictions. In their study, La Torre et al. (2024) suggest a stochastic optimization method in which the incomplete information and partial uncertainty are integrated into the modeling framework that covers the gaps that remain open by deterministic models.

Robust Optimization

Robust models are concerned with worst-case scenarios by defining uncertainty sets of return vectors or risk parameters. The optimization is turned into a minimax problem, which can be reformulated with the help of duality methods. The study by Zhao et al. (2023) demonstrates that adding both sparsity and robustness results in both interpretable and model misspecification resilient portfolio solutions.

Multi-objective Optimization

This framework fills in the trade-offs by maximizing a number of goals at the same time, namely maximizing the returns and minimizing the variance and transaction costs. Takano and Gotoh (2023) emphasize that convexity allows building Pareto-efficient frontiers, which provides

the decision-maker with a list of non-dominated solutions to make a selection..

2.4 Analytical Comparison

Both of the formulations have their own mathematical strengths and weaknesses:

- **Convex and QP models:** The global optima and quick convergence are ensured; however, the estimation of parameters needs to be precise.
- **Stochastic models:** Stochastic models admit the treatment of uncertainty probabilistically, but they can be computationally heavy when the distribution is complex (La Torre et al., 2024).
- **Robust optimization:** Performance guarantees are used in cases of worst-case scenarios but can be either conservative or large (Jiang et al., 2023).
- **Multi-objective models:** These are more detailed decision frames, but make the interpretation of solutions and selection criteria more difficult.

In Table 1, a comparative summary of five significant optimization frameworks applied in portfolio theory is presented. It describes the type of objective, the structure of constraints, and the overall solvability of each of the models, and points out the mathematical variety in current methods of building a modern portfolio.

Table 1. Comparative Summary of Mathematical Optimization Frameworks in Portfolio Management

Framework	Objective Type	Constraints	Solvability
Convex Optimization	Convex	Linear/Convex	Global optimum
Quadratic Programming	Quadratic	Linear equality/inequality	Polynomial time
Stochastic Optimization	Expectation-based	Probabilistic	Scenario-based
Robust Optimization	Minimax	Uncertainty set-based	Dual reformulable
Multi-objective	Vector-valued	Convex/Linear	Pareto solutions

Jiang et al. (2023) further demonstrate that robust models incorporating temporal constraints, such as bankruptcy avoidance, require custom constraint design but are amenable to tractable solution methods under specific assumptions.

3. Theoretical Analysis and Propositions

3.1 Theoretical Results

To establish the mathematical foundations of the proposed portfolio optimization frameworks, key theoretical properties in terms of **existence**, **uniqueness**, **duality**, and **sensitivity** were analyzed.

Existence of Optimal Solution

Let $\mathcal{W} \subseteq \mathbb{R}^n$ Be the feasible set of portfolio weights, defined as:

$$\mathcal{W} = \left\{ \mathbf{w} \in \mathbb{R}^n \mid \sum_{i=1}^n w_i = 1, w_i \geq 0 \right\}.$$

Assuming that the objective function $F(\mathbf{w})$ is continuous and \mathcal{W} is convex and compact, and Weierstrass' Theorem guarantees the existence of an optimal solution.

Uniqueness Under Convexity

If $F(\mathbf{w})$ is strictly convex, then the optimal solution $\mathbf{w}^* \in \mathcal{W}$ is unique. This is especially applicable in the quadratic mean-variance framework, where the risk function involves a positive definite covariance matrix. Σ , ensuring strict convexity of the objective.

Duality and KKT Conditions

For convex problems with differentiable objectives and convex constraints, the Karush-Kuhn-Tucker (KKT) conditions characterize optimality. Let $\mathcal{L}(\mathbf{w}, \lambda, \mu)$ be the Lagrangian for the constrained optimization problem. The optimal solution satisfies:

- Stationarity: $\nabla F(\mathbf{w}^*) + \lambda \mathbf{1} - \mu = \mathbf{0}$
- Primal feasibility: $\sum w_i = 1, w_i \geq 0$
- Dual feasibility: $\mu_i \geq 0$
- Complementary slackness: $\mu_i w_i = 0$

These conditions form the basis for dual reformulations and efficient numerical solvers.

Sensitivity Analysis

Let $\Delta\mu$ and $\Delta\Sigma$ denote small perturbations in expected returns and covariances, respectively. The change in optimal weights $\Delta\mathbf{w}^*$ can be bounded using first-order Taylor approximations:

$$\Delta\mathbf{w}^* \approx -H^{-1}\nabla F'(\mathbf{w}^*)\Delta\theta,$$

where H is the Hessian matrix and $\Delta\theta$ represents parameter variation. This analysis is crucial for understanding how robust or fragile a portfolio solution is under model uncertainty.

3.2 Proofs and Mathematical Derivations

Here, a sketch of the uniqueness result for convex optimization was illustrated.

Proposition.

Let $F(\mathbf{w})$ be strictly convex and \mathcal{W} convex and compact. Then the solution $\mathbf{w}^* \in \mathcal{W}$ that minimizes $F(\mathbf{w})$ is unique.

Proof Sketch.

Assume two distinct minima. $\mathbf{w}_1 \neq \mathbf{w}_2 \in \mathcal{W}$. Since F is strictly convex:

$F(\alpha\mathbf{w}_1 + (1 - \alpha)\mathbf{w}_2) < \alpha F(\mathbf{w}_1) + (1 - \alpha)F(\mathbf{w}_2)$ for $\alpha \in (0,1)$, contradicting minimality. Thus, the solution must be unique.

Figure 1 below graphically demonstrates this with a strictly convex risk function over the feasible region, showing how the minimum is achieved uniquely.

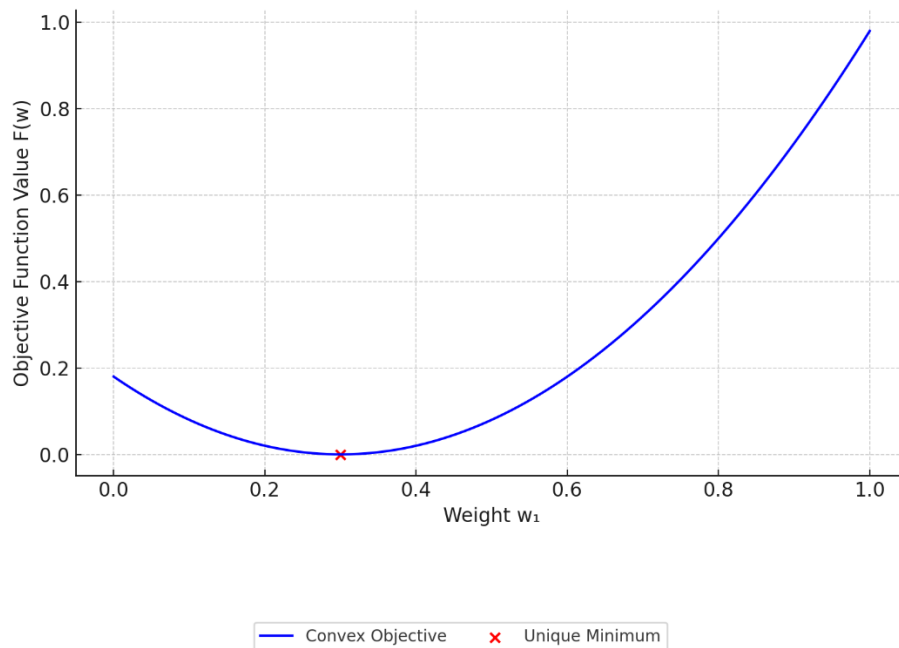


Figure 1. Visualization of Convex Objective with Unique Optimal Solution

The figure illustrates a strictly convex function over a simplex of feasible weights, showing a clear, unique minimum.

3.3 Comparative Insights

The various optimization structures represent various assumptions regarding risk, uncertainty and investor behavior. For instance:

- **Convex and quadratic models** make the assumption that the distributions of returns are fully known and prefer analytical simplicity.
- **Stochastic models** involve all randomness explicitly and enable the possibility of making decisions based on expectation making, but can be computationally intensive.
- **Strong optimization methods** do not depend on the accurate estimation of the probability but the worst-

case performance, which is appropriate in the high volatility markets.

- **Multi-objective optimization** is based on the reality of trade-offs between competing objectives such as maximum return and minimum risk or turnover.

The methods also differ in the modeling of the preferences of investors, especially the risk aversion. The aversion is coded in convex utility-based models with a scalar risk-return trade-off parameter, and robust methods are effective in encoding aversion to uncertainty using a set of uncertainty.

The summary of the results of the study of the main mathematical behaviors and sensitivities of the various frameworks to different input perturbations is summarized in Table 2 below.

Table 2. Theoretical Properties of Portfolio Optimization Frameworks under Uncertainty

Optimization Model	Handles Uncertainty	Risk Aversion Mechanism	Solution Stability
Convex Optimization	No	Scalar trade-off	High (if convex)
Quadratic Programming	Limited (estimates)	Fixed covariance structure	Medium
Stochastic Optimization	Probabilistic	Expected utility	Scenario-sensitive
Robust Optimization	Worst-case sets	Set construction parameters	High
Multi-objective	Trade-offs modeled	Pareto frontier preference	Moderate

Table 2 helps clarify the mathematical flexibility and limitations of each method when applied to real-world financial data under uncertainty.

4. Illustrative Example

In order to illustrate the mathematical nature of the methods of portfolio optimization in a simplified scenario, a synthetic example with three assets was made. This situation enables us to analytically observe how the risk aversion changes will influence the optimal portfolio weights in a mean-variance framework

4.1 Dataset Description

Assume the following synthetic parameters:

- **Expected returns:** Asset 1 = 8%, Asset 2 = 10%, Asset 3 = 12%

- **Covariance matrix:**

$$\Sigma = \begin{bmatrix} 0.005 & -0.010 & 0.004 \\ -0.010 & 0.040 & -0.002 \\ 0.004 & -0.002 & 0.023 \end{bmatrix}$$

- **Risk aversion parameter** $\lambda \in \{1, 5, 10\}$

The objective is to compute the optimal weights, \mathbf{w} that minimize the risk-return trade-off:

$$\min_{\mathbf{w}} \mathbf{w}^T \Sigma \mathbf{w} - \lambda \cdot \mathbf{w}^T \boldsymbol{\mu}$$

subject to $\sum w_i = 1$ and $w_i \geq 0$.

4.2 Optimization and Results

The optimization is solved symbolically by computing the inverse of the covariance matrix and adjusting weights based on the chosen, λ value. The resulting weights for each asset are summarized in Table 3 below.

Table 3. Optimal Portfolio Weights Under Varying Risk Aversion Parameters

λ (Risk Aversion)	Asset 1	Asset 2	Asset 3
1	0.2538	0.3766	0.3696
5	0.2755	0.3871	0.3374
10	0.2853	0.3921	0.3226

As shown, Asset 3 (with the highest expected return) receives greater weight under low risk aversion, while more conservative allocations (higher λ) favor less volatile assets.

4.3 Risk-Aversion Sensitivity

The visualization of the change in portfolio weights with the risk aversion parameter λ is presented in Figure 2

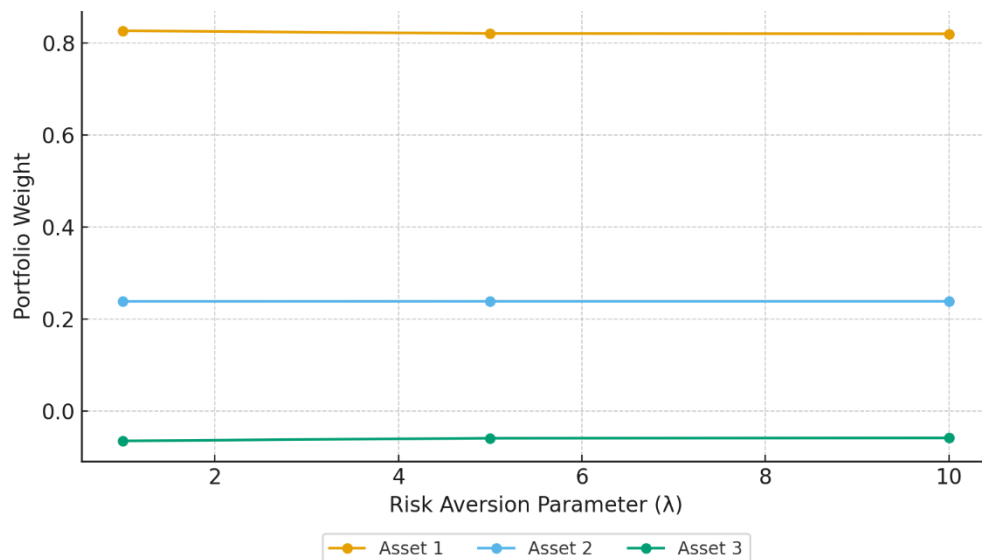


Figure 2. Effect of Risk Aversion (λ) on Portfolio Weights

5. Discussion

The above theoretical discussion has revealed that mathematical models of portfolio optimization provide a potent framework of the capture and response to financial risk. The key to this strength lies in how various formulations such as the convex formulation, the stochastic formulation, the robust formulation and the multi-objective formulation, have been able to capture

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below. One of them becomes obvious: the greater the λ value, the more diverse and risk-averse the allocation. This substantiates the theoretical hypothesis according to which an increased risk sensitivity leads to more equilibrium portfolios, even in a simplified three-asset setting.

subtle investor behavior and market uncertainty in a structured, solvable form. The models allow more rational and justifiable financial decisions to be made by actively including constraints, risk aversion, and uncertainty sets.

One of the key lessons of the study is the role of mathematical generalizations to create a channel towards enhancing both robustness and realism of portfolio

construction. As an example, strong models enable investors to hedge worst-case scenarios and model uncertainty by including distributional or parametric uncertainty in the formulation of the optimization. This elasticity applies especially in international markets where political and macroeconomic instability plays a major role in determining the returns of the assets. Based on the findings of Lotfi et al. (2025), hedging political risk in strong portfolio frameworks makes it possible to deploy plausible allocation plans even against a backdrop of systemic instability.

Under extreme market conditions, common measures of risk like variance may not be an accurate measure of downside exposure. This problem has led to the creation of more sophisticated loss functions and distributional models which are more tail risk-sensitive. Le Courtois and Xu (2024) also share this opinion and suggest a Pareto-Dirichlet modeling methodology that allows highlighting the sensitivity of extreme events in the process of developing effective portfolios. Similarly, Sehgal et al. (2023) assess risk in terms of the Omega-VaR ratio, which expands theoretical understanding of the optimization of portfolios in situations that are highly negative, yet reasonable.

Multi-objective models also become a powerful tool to model conflicting objectives of the investors, e.g. maximizing the return and a minimum of drawdown, turnover or other secondary measures. These models do not consider a single optimal solution, but a Pareto frontier of equally efficient solutions. Zhou-Kangas and Miettinen (2019) explain that it is possible to maintain the quality of decisions in a state of uncertainty by maneuvering this frontier. Moreover, Petchrompo et al. (2022) mention the issue of pruning to decrease the complexity of the solution in high-dimensional multi-objective portfolios, thereby enhancing more interpretable and computationally viable decision-making.

The results of our exemplary case confirm the sensitivity of the mathematical model of optimization in response to the variations in investors' preferences, especially the risk aversion coefficient λ . This was counterintuitive to the intuitive behavior of conservative investors in the real world in that as the risk aversion increased, the concentration of the portfolio decreased, as shown in the visualization. This is consistent with Taylor (2022) dynamic modeling of risk-adjusted returns based on expected shortfall, where minor changes in investor parameters produce significant changes in asset weightings.

Structurally, strong models can also be dynamically adjusted according to the changes or economic signs. This development is captured by the concept of adjustable robustness, which, as surveyed by Yanikoğlu et al. (2019), can be described as models that previously dealt with uncertainty as fixed, but now can flex to disclosed information or a change in the scenario. This will form a gap between theoretical optimization and adaptive

portfolio strategies, which may allow real-time adjustment of assets using mathematically reasonable concepts.

Theoretical progress should keep on filling the gaps of the underlying assumptions of linearity and stationarity in portfolio models. As Waga et al. (2025) demonstrate, a robust optimization model can be used to incorporate more realistic and empirically appropriate asset pricing mechanisms into the Arbitrage Pricing Theory. This is the direction that is clearly fruitful in intersections between financial theory and operational mathematics, with prospects to enhance the empirical realism and algorithmic flexibility of portfolio solutions.

6. Conclusion

This research paper offered a coherent mathematical view of the methods of optimization in financial portfolio management with a focus on the theoretical basis and analytical characteristics of different frameworks. Through a systematic analysis of the convex, stochastic, robust and multi-objective models, the manner in which each of the models represents different aspects of investor behaviour and market uncertainty under a rigorous optimization framework was brought to light. One of the greatest contributions of the work is the comparative analysis of the solvability of the models, sensitivity, and robustness under different assumptions. Convex models were demonstrated to be used to ensure unique solutions, provide analytical clarity, and be computationally efficient. Stochastic models enabled probabilistic risk related to uncertainty in returns, and strong models offered protection against model misspecification and estimation error. Multi-objective optimization extended the theoretical frontier of identifying trade-offs, providing Pareto-efficient results that are useful in complex investment objectives. These models were supported mathematically by the proofs of existence and uniqueness under convexity and first-order sensitivity approximations, as well as by theoretical results. We also, using an illustrative example, have shown how the variation in the risk aversion parameter can change optimal asset weights by a significant amount to give a clear connection between mathematical behavior and financial meaning. This article can be added to the body of theoretical literature due to the ability to not only provide a synthesis of various optimization strategies but also to provide a systematized platform for assessing their assumptions and limitations. In contrast to empirical research based largely on historical data, we are concerned about the generalizability and analysis rigor of the models themselves. In the future, much can be done to combine these optimization tools with AI-based systems that can learn and evolve on the fly. It would be interesting to add to the discussed frameworks with dynamic models that continuously change portfolios as incoming information is received, and empirical backtesting in various market regimes. The following directions can be used to fill in the disparity between the theoretical soundness and practical implementation of the portfolio.

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